The 5th Italian-Japanese Workshop on Geometric Properties for Parabolic and Elliptic PDE's

Abstract

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On the rate of convergence in free boundary problems

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We discuss the rate of convergence of approximate solutions via penalization in free boundary problems. As a typical example, we consider uniformly elliptic equations with gradient constraints, which arise in singular stochastic control problems:

$$\max\{-\Delta u + u - f, |Du| - 1\} = 0 \quad \text{in } \Omega, \tag{1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain, and $f \in C^2(\overline{\Omega})$ a given function. In [1], Evans showed the existence of strong solutions in $W^{1,\infty}(\Omega) \cap W^{2,\infty}_{loc}(\Omega)$ of (1) under Dirichlet condition. To this end, we estimate approximate solutions of the associated penalty equations. Afterwards, Ishii-K in [3] obtained $W^{2,\infty}(\Omega)$ estimates on these under some additional assumptions. In order to establish the rate of convergence of approximate solutions to that of (1), we use some estitames in [3] combining nonlinear adjoint method, which was introduced by Evans in [2].

This is a joint work with Dr. T. Kosugi and M. Naito.

References

[1] L. C. Evans, Comm. Partial Differential Equations, 4 (5), 555-572, (1979)

[2] L. C. Evans, Arch. Ration. Mech. Anal., 197 (3), 1053-1088, (2010)

[3] H. Ishii and S. Koike, Comm. Partial Differential Equations, 8 (4), 317-346, (1983)

Lipschitz stability in some inverse problems

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One important issue in the study of inverse problems is stability, that is the way the reconstruction deteriorates in the presence of errors in the measurements. In this talk I will show that inverse boundary value problems need strong a priori assumptions to ensure a good rate of stability. I will also talk about some hybrid inverse problems that show a better rate of stability.

Optimization problems related to some anisotropic operators

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In this talk we will describe some recent results on sharp bounds, in terms of quantities which depend on the geometry of the domain, for the extrema of certain functionals which involve an anisotropic norm of the gradient. The results we will described are obtained in collaboration with F. Della Pietra, S. Guarino Lo Bianco and G. Piscitelli.

On Liouville properties of Riemannian manifolds

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When any L^p-harmonic function on a Riemannian manifold M is trivial we say that M enjoys the L^p-Liouville property. We will show how the L^2-Liouville property is related with the essential self-adjointness of the Laplacian, and we will discuss the L^1-Liouville property in terms of the behavior of Brownian motion. This is a joint work with A. Grigoryan and M. Murata.

Fast Diffusion Equations and its Applications

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In this talk, we are going to discuss discrete velocity Boltzmann type equation of a fictitious gas proposed by Carleman and its diffusive hydrodynamic limit to fast diffusion or porous medium type, $u_t = \Delta u^m$ $(0 \leq m \leq 2)$ including log-diffusion equations in higher dimensions. The main difficulty is the lack of mass conservation, which has been overcome by nontrivial construction of super- and sub-solutions correcting the error in each small scaling parameter.

We will also discuss how to apply log-diffusion equation to get the geometric properties of the nonzero minimal solution of Gelfand Problem.

Finsler heat equation with growing initial data

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We obtain an optimal growth condition at the space infinity on the initial data for which the Cauchy problem to the Finsler heat equation

$$\partial_t u = \Delta_H u$$

possesses a local-in-time solution. This is a joint work with Goro Akagi (Tohoku University, Japan) and Ryuichi Sato (Tohoku University, Japan).

The porous medium equation with large initial data on negatively curved Riemannian manifolds

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We discuss existence and uniqueness of very weak solutions to the Cauchy problem for the porous medium equation on complete, simply connected Riemannian manifolds with nonpositive sectional curvatures (also known as Cartan-Hadamard manifolds), which in addition are supposed to satisfy suitable bounds from below on Ricci curvature. Having in mind the pioneering paper by Bénilan, Crandall and Pierre on Euclidean space, we allow the initial datum to have a precise power-type growth rate at spatial infinity. Such a rate turns out to depend crucially on the curvature bounds. For instance, the associated pressure is allowed to grow at most linearly on hyperbolic space and quadratically on Euclidean space, as well as on a class of manifolds whose Ricci curvature vanishes sufficiently fast at infinity. The curvature conditions we require are to some extent optimal: if they are not met then uniqueness fails even for bounded initial data. Moreover, upon assuming upper bounds on sectional curvatures that match the lower ones, we can give a sharp estimate for the maximal existence time of a solution, which shows in particular that our growth hypotheses on the initial datum are in turn sharp. Finally, at such maximal existence time, we can provide precise pointwise blow-up results at least for a particular class of manifolds and initial data that fit within our framework.

On the solution structure of bistable reaction-diffusion equations on a thin dumbbell-shaped domain

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In this talk, we discuss the dynamics of solutions of scalar bistable reaction-diffusion equations with the Neumann boundary condition. We particularly focus on the case that the domain is dumbbell-shaped and close to a line segment. Then the equation can be approximated by some one-dimensional limiting equation. After introducing what the limiting equation should be, we show that the limiting equation indeed approximates the original equation, and then we discuss the solution structure of the limiting problem.

Loss of energy concentration in nonlinear evolution fourth order equations

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We consider a class of nonlinear evolution beam equations for which we define a notion of prevailing mode, inspired by the oscillations observed at the Tacoma Narrows Bridge. We then introduce a new definition of instability for solutions with a prevailing mode, embodying a significant and sudden loss of energy concentration from the prevailing mode onto another one. We reduce the search for stability and instability for the problem under consideration to the study of a suitable finite-dimensional approximation, and we show some numerical simulations. We also consider the case of a multiple beam divided by intermediate piers and discuss some possible extensions. Joint works with Filippo Gazzola (Polytechnic University of Milan).

Diffusion phenomena for the wave equation with space-dependent unbounded damping term

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We discuss the asymptotic behavior of solutions to the wave equation with damping depending on the space variable and growing at the spatial infinity. If the damping coefficient is positive constant, then it is well-known that the asymptotic behavior can be given by a solution to heat equation. Similar phenomenon occurs when the damping coefficient decays slowly at the spatial infinity. In this talk we focus our attention to the case where damping coefficient is growing at spatial infinity. The crucial point of this study is to construct a weighted energy estimate for wave equation with damping and a weighted L^p-L^q estimate for the corresponding parabolic equation.

Variational and non-variational solutions for a p-Laplace eigenvalue problem

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We consider the eigenvalue problem for the p-Laplace operator that arises in the minimisation of the p-homogeneous Dirichlet energy with a L^q constraint, where $1 < q < p^*$. We discuss some analogies and differences with the linear theory; in particular, we present the example of an open set on which the first eigenvalue is not isolated and the spectrum is not discrete, so that the topological approach to the global analysis of nonlinear eigenvalue problems does not detect the whole spectrum. The results are obtained in collaboration with Lorenzo Brasco (University of Ferrara, Italy).

Two-phase heat conductors in two dimensions and related overdetermined problems

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In the paper [S], we considered a two-phase heat conductor in $\mathbb{R}^N$ with $N \geq 2$ consisting of a core and a shell with different constant conductivities. Among other things, when the medium outside the two-phase conductor has a possibly different conductivity, we treated the Cauchy problem for $N \geq 3$ with the initial condition where the conductor has temperature 0 and the outside medium has temperature 1. It was shown in [S, Theorem 1.3, p. 168] that if there is a stationary isothermic surface in the shell near the boundary, then the structure of the conductor must be spherical. Because of some technical reasons, the case where $N = 2$ was excluded. Here we show that the same proposition holds true even when $N = 2$, and we also treat a similar overdetermined problem where a stationary isothermic surface is replaced by a surface with the constant heat flow property.

References

[S] S. Sakaguchi, Two-phase heat conductors with a stationary isothermic surface, Rendiconti dell'Istituto di Matematica dell'Università di Trieste, 48 (2016), 167–187.

Torsion function, homogeneous Sobolev spaces and negative trapping potentials

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Let Ω be a generic open set of the Euclidean space. We consider the *homogeneous Sobolev space*, defined by the completion of the space of compactly supported smooth functions with respect to the L^2 norm of the gradient.

We give a characterization of the continuous (or compact) embedding of this space into $L^q(\Omega)$, in terms of the summability of the so-called *torsion function* of Ω. We also introduce a new Hardy-type inequality, which plays an important role in the proofs.

Finally, we give some applications of our Hardy-type inequality to ground state estimates for Schrödinger operators with negative trapping potentials.

The results presented are contaied in joint works with Giovanni Franzina (Roma) and Berardo Ruffini (Montpellier).

A limit equation and bifurcation diagrams of semilinear elliptic equations with general supercritical growth

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We study radial solutions of the semilinear elliptic equation $\Delta u+f(u)=0$ under rather general growth conditions on f. We construct a radial singular solution and study the intersection number between the singular solution and a regular solution. An application to bifurcation problems of elliptic Dirichlet problems is given. To this end, we derive a certain limit equation from the original equation at infinity, using a generalized similarity transformation. Through a generalized Cole-Hopf transformation, all the limit equations can be reduced into two typical cases, i.e., $\Delta u+u^p=0$ and $\Delta u+e^u=0$.

Best constant in Poincaré and Sobolev inequalities with trace term

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The question of finding Sobolev inequalites with trace terms was raised by Brézis and Lieb in 1985 and takes the form: given $\alpha, \beta > 0$ and $p, q, r \geq 1$, prove that

$$\|\nabla u\|^{\alpha}_{L^p(\Omega)} + \beta \|u\|^{\alpha}_{L^q(\partial\Omega)} \geq C \|u\|^{\alpha}_{L^r(\Omega)}$$

for all measurable functions u and for all bounded Lipschitz domains $\Omega \subset R^N$, with a constant C depending only on β, N and the measure of Ω. One of the main points is to find the best constant C and, if true, to prove it corresponds to Ω being a ball.

This question was solved by Maggi and Villani in the case $p \in [1, N)$, $q = \frac{(N-1)p}{N-p}$, $r = \frac{Np}{N-p}$, by Bossel and Daners in the case $\alpha = p = q = r = 2$ and by Dai and Fu in the case $\alpha = p = q = r \in (1, +\infty)$ (in the family of Lipschitz domains). The last two results are also related to the Faber-Krahn inequality for the Laplace (p-Laplace) operator with Robin boundary conditions.

In this talk, I will discuss a series of recent results obtained with A. Giacomini allowing to treat a full class of such inequalities, for arbitrary domains and less regular functions. The key point is related to the regularity analysis of solutions of a family of free discontinuity problems.

Dynamics of solutions of the Fisher-KPP equation for slowly decaying initial data

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This talk is concerned with the dynamics of solutions of the Fisher-KPP equation when initial data decay slowly in space. We first show that the large-time behavior of solutions is determined by the decay rate of initial data. Next, we show that in a certain singular limit, the motion of the interface can be described as a level set of a first-order PDE of Hamilton-Jacobi type. Then by studying relationship between the Fisher-KPP equation and the Hamilton-Jacobi equation, we can track the long-time behavior of interfaces. As a consequence, we can show the existence of expanding interfaces with an arbitrary shape. This is a joint work with Hirokazu Ninomiya of Meiji University.

A comparison principle for singular degenerate parabolic equations under some dynamic boundary conditions

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In the context of viscosity solutions, a comparison principle for dynamic boundary value problems was proved by Barles (1993, 1999) provided that there is no singularity in equations. However, the comparison principle has not been known for singular equations such as the mean curvature flow equation. In this talk, under some simplified situations, we establish a comparison principle for viscosity solutions to singular degenerate parabolic equations with dynamic boundary conditions. In the proof, we add an extra parameter to a test function in order to give a precise study of singularities. This talk is based on a joint work with Professor Yoshikazu Giga (The University of Tokyo).

Isoperimetric inequalities for factorized measures

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It is well known that the isoperimetric sets for the gaussian and the anti-gaussian measure, $d\mu_{\pm} = \exp\left(\pm\frac{|x|^2}{2}\right) dx$, are half-spaces and balls centered at the origin, respectively. In this talk we will discuss the isoperimetric problem for factorized measures obtained as perturbations of $d\mu_+$ and $d\mu_-$. (Joint works with F. Brock and A. Mercaldo)

Hadamard variational formula for the eigenvalue of the Stokes equations and its applications

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We consider the domain perturbation problem of the Stokes equations, which describes the motion of the incompressible viscous fluid moving slowly. Jimbo-Ushikoshi (2015) established the first variational formula for the multiple eigenvalue of the Stokes equations with the Dirichlet boundary condition under a smooth domain perturbation, which is so called Hadamard variational formula. In this talk, we clarify the topological type of the domain in 3D by that formula. This work is a joint work with Prof. Shuichi Jimbo (Hokkaido Univ.), Hideo Kozono (Waseda Univ.) and Yoshiaki Teramoto (Setsunan Univ.).

On the stability for Alexandrov's Soap Bubble theorem

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Alexandrov's Soap Bubble theorem dates back to 1958 and states that a compact hypersurface embedded in $\mathbb{R}^N$ with constant mean curvature must be a sphere.

In this talk, based on some integral identities, we will discuss about the stability of the spherical symmetry: the question is how much a hypersurface is near to a sphere, when its mean curvature is near to a constant in some norm.

We will present a stability estimate that states that a compact hypersurface $\Gamma \subset \mathbb{R}^N$ can be contained in a spherical annulus whose interior and exterior radii, say ρ_i and ρ_e, satisfy the inequality

$$\rho_e - \rho_i \leq C\|H - H_0\|_{L^1(\Gamma)}^{\tau_N}$$

where $\tau_N = 1/2$ if $N = 2, 3$, and $\tau_N = 1/(N+2)$ if $N \geq 4$. Here, H is the mean curvature of Γ, H_0 is some reference constant and C is a constant that depends on some geometrical and spectral parameters associated with Γ. This estimate improves previous results in the literature under various aspects.

We will also deal with similar stability estimates for some related overdetermined boundary value problems.

This talk is based on a joint work with Rolando Magnanini (University of Florence).

On the existence of maximizers for inhomogeneous Trudinger-Moser type inequalities on the whole space

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We discuss the existence or non-existence of maximizers for Trudinger-Moser type inequalities of inhomogeneous type. Trudinger-Moser inequality was originally obtained by Trudinger and Moser in bounded domains as the limiting case of Sobolev embeddings. After that, several authors succeeded in extending the inequality to unbounded domains. Especially, Ruf (2005)and Li-Ruf (2008) established inhomogeneous Trudinger-Moser type inequalities in the whole space with best constants. In this talk, we will consider the existence and non-existence of these type inequalities. The difficulties come from a lack of compactness of the corresponding functional due to concentrating and vanishing phenomena. We shall give sufficient conditions on the exponents appearing in the inequality so that its maximizers exist or not. This is a joint work with Professor Norihisa Ikoma in Kanazawa University and Michinori Ishiwata in Osaka University.

Maximizers for the Trudinger-Moser inequality on the whole plane with the critical growth

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This is joint work with Slim Ibrahim, Nader Masmoudi and Federica Sani. The Trudinger-Moser inequality is a substitute for the forbidden critical Sobolev embedding on the plane, giving a uniform bound on the square exponential integral in bounded domains, instead of a pointwise uniform bound. It is known that the original form of the inequality with the sharp exponent fails on the whole plane even after a renormalization, but a modified version is obtained with an exactly critical growth condition, which is slightly slower than the square exponential. In this talk, I would like to discuss about existence of maximizers for the sharp inequality. This problem is intimately related to existence of the ground state solitons, which play crucial roles in the global dynamics for the evolution equations associated with the energy. For the original Trudinger-Moser inequality on the disk, its attainability was proved by Carleson and Chang. It turns out that the whole space version is more critical in the sense that small order perturbations can change the attainability around the nonlinearity with the exact growth. More precisely, when we add small order perturbation for large values of the nonlinearity, maximizers exist if the perturbation is positive, but if it is negative, we have no maximizer. The same things happen when small values of the nonlinearity is perturbed. This phenomenon is similar to that in the result of Brezis and Nirenberg for the critical Sobolev embedding in higher dimensions, but the two dimensional case looks more complicated, since the concentration of energy is not simply by the invariant dilation. In fact, the key in the proof is to decompose maximizing sequences into the concentrating part and the spreading part, with precise asymptotic description of each.

New Pólya-Szegö-type inequalities

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Some Pólya-Szegö-type inequalities which involve couples of functions and their rearrangements are presented. These inequalities reduce to the classical Pólya-Szegö principle when the two functions coinccide. As an application it is given a different proof of a well-known comparison result for solutions to Dirichlet boundary value problems for Laplacian equations with respect to Steiner symmerization.

Poster session abstract

Locally optimal configurations for the two-phase torsion problem in the ball

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We consider the unit ball filled with two materials with different conductivities. We perform shape derivatives up to the second order to find out precise information about locally optimal configurations with respect to the torsional rigidity functional. Depending on the various parameters, a symmetry breaking phenomenon occurs.

Classification of radial solutions to Hénon type equation on the hyperbolic space

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We consider classification of radial solutions to a weighted semilinear elliptic equation on the hyperbolic space. In particular, choosing an appropriate weight, we shall prove that the weighted equation has rich structure of solutions, such as the existence of solutions with infinitely many zeros and critical exponents on the sign of solutions.

On the elliptic equation with the Hardy-Sobolev critical exponent

Masato Hashizume (Osaka City University)

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We study existence and non-existence of ground state solution of an elliptic equation with the Hardy-Sobolev critical exponent. We consider impact of the mean curvature at singularity and scale of domain. In addition, we also study the uniqueness of ground state solution.

Stability analysis on a hybrid PDE-ODE system modeling intermittent hormonal therapy

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We consider a control problem by hybrid PDE-ODE systems modeling intermittent hormonal therapy of prostate cancer. Since the purpose of hybrid systems is not only minimization or maximization of a cost functional, it is not clear what is optimal in control prescribed by hybrid systems. In our poster, we shall give a concept of stability of the control. Moreover, we show sufficient conditions on initial data for the existence of the stable control.

Solvability of the heat equation with a nonlinear boundary condition

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We consider necessary conditions and sufficient conditions for the solvability of the heat equation with a nonlinear boundary condition. This is a joint work with professor K. Ishige (Tohoku University).

A singular perturbation limit of diffused interface energy with a fixed contact angle condition

Takashi Kagaya (Tokyo Institute of Technology)

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We consider a general asymptotic behavior of critical points of a diffused interface energy with a fixed contact angle condition defined on a bounded domain. We study the contact angle of the limit varifold derived from the diffused energy under a set of assumptions.

Maximum principle for Pucci equations with sublinear gradient terms and its applications

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In 1989, Caffarelli established the regularity theory for viscosity solutions of fully nonlinear uniformly elliptic equations. The Alexandrov-Bakelman-Pucci (ABP for short) and the weak Harnack inequality play important roles in his regularity theory. In this session, we consider the ABP maximum principle and the weak Harnack inequality for Pucci equations with sublinear growth in Du. As an application, we obtain the Hölder continuity of viscosity solutions of singular elliptic equations involving p-Laplace equations.

Asymptotic expansions of solutions of fractional diffusion equations

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We obtain the precise description of the asymptotic behavior of the solution to the linear fractional diffusion equation with an weighted L^1 initial data. Furthermore, we establish a method to obtain the asymptotic expansions of the solutions to inhomogeneous fractional diffusion equations and nonlinear fractional diffusion equations. Joint work with K. Ishige and T. Kawakami.

A singular perturbation approach to elastic curve problems

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In general, it is difficult to know the shapes of elastic curves for arbitrarily given boundary conditions. In this presentation, for a generic boundary condition, we reveal the asymptotic shape of elastic curves as the effect of bending rigidity tends to be zero. Our main tool is singular perturbation theory.

Strauss's radial compactness and its application to nonlinear elliptic problem with variable critical exponent

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We consider on compactness for the embedding from radial Sobolev spaces $W^{1,p}_{\mathrm{rad}}(\mathbb{R}^N)$ to variable exponent Lebesgue spaces $L^{q(x)}(\mathbb{R}^N)$. As an application we prove the existence of solutions of the quasi-linear elliptic equation with a variable critical exponent.

Two-phase eigenvalue problem on thin domains with Neumann type boundary condition

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We consider an eigenvalue problem with piecewise constant coefficients on thin domains with Neumann type boundary condition, and we analyze the asymptotic behavior of each eigenvalue as the domain degenerates into a certain hypersurface being the sets of discontinuities of the coefficients. We show how the discontinuity of the coefficients and the geometric shape of the interface affect the asymptotic behavior of the eigenvalues by using variational approach.